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THE COST OF EQUITY CAPITAL IN STOCK PORTFOLIOS LISTED ON THE WARSAW STOCK EXCHANGE USING THE CLASSIC CAPM

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Abstract

This work is an attempt to estimate the cost of equity capital characteristic among portfolios of companies listed on the Warsaw Stock Exchange in the years 1995-2017. To this end, the classic CAPM is used to estimate the cost of risk. Model tests are based on 252 monthly returns. In order to assess the errors of cost of capital estimation, the bootstrap method is used. The estimated cost of capital refers to the project portfolio with real options on these projects. Stock returns are generated not only by the companies implementing projects but also through real options modifying these projects. The estimated cost of capital can be a valuable indicator for portfolio managers. Also, it can be an approximate indicator for making decisions on the implementation of new investment projects. The estimated cost of capital assumes the highest values for value portfolios. The estimated cost of capital assumes the small values for growth portfolios.

JEL classification: G11, G12

Keywords: ICAPM, Cost of capital, Risk premium

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Introduction

The classic CAPM, proposed by Sharpe (1964) and Lintner (1965) outlines the theoretical basis for investment decisions of companies. Literature evidence confirms the applicability of CAPM to the cost of capital estimate and assessment of the effectiveness of investment projects. Examples include the work of Graham and Harvey (2001) or another work of Welch (2008). In the first work the authors state that the classic CAPM, average stock return, and multifactor CAPM are the most popular methods, respectively, for assessment of the cost of capital. A dividend discount model is the least used. Also, the authors point out that although CAPM is the most popular "... it is not clear that the model is applied properly in practice." (Graham & Harvey, 2001, p. 201). This can be a reason for pricing anomalies connected with size or book to market value (BV/MV) effects (Banz, 1981; Rosenberg, Reid & Lanstein, 1985 or Fama & French, 1992). Other anomalies disturbing pricing in light of the CAPM are examined, among others by Lakonishok, Shleifer and Vishny (1994) or Jegadeesh and Titman (1993). The results of other works of Reinganum (1981) and Lakonishok and Shapiro (1986), confirmed by Fama and French (1992) state that the risk should be seen in a multidimensional space. However, in the above-mentioned work, Welch (2008) recommends the CAPM much more widely than the more theoretically advanced ICAPM or APT applications².

Jagannathan and Wang (1996), Berk, Green and Naik (1999), and Bernardo, Chowdry, and Goyal (2007), and Zhi, Guo and Jagannathan (2012) present an attempt to explain the incompatible pricing that could be observed in the conditions of CAPM validity. The authors agree that companies often secure intended and implemented projects by real options on these projects. Therefore, the company can be seen as a portfolio of current and future projects as well as complex options on these projects.

Based on the above short literature analysis it can be concluded that if all company projects are not influenced by real options, then the necessary and sufficient condition for estimating the cost of equity capital by classic CAPM or ICAPM applications is the use of a pricing application consistent with the CAPM or ICAPM.

 2 Welch (2008, p. 1) states: "75% of finance professors recommend using the CAPM for corporate capital budgeting purposes; 10% recommend the Fama-French model; 5% recommend an APT model."

Therefore, it seems that the estimation of the cost of capital will be more correct for assets resistant to price anomalies. Assuming that pricing of assets in light of the CAPM is found for portfolios, it seems advisable to make an attempt to estimate the cost of capital of specific stock portfolios on a given market. Ferson and Locke (1998) show that the necessary condition for correct estimation of the cost of capital using CAPM is proper estimation of the risk premium which is much more important than inaccurately estimated systematic risk. Thus, precise estimate of the risk premium, and pricing that could be observed in conditions of the CAPM validity, require using a pricing application that allows you to generate meanvariance-efficient portfolios.

The aim of this study is to estimate the cost of capital (median value and corresponding confidence interval) of characteristic portfolios of companies, using the classic CAPM, and assuming that the investment projects implemented by the company are independent of each other and the impact of real options is negligible. This work presents methodologies that constitute the basic research of estimating the cost of capital. The study is based on stocks listed on the Warsaw Stock Exchange (WSE). However, this work differs from the procedures of risk premium estimation adopted so far. In this study, the risk premium is estimated on the basis of the econometric model presented in the next section. The use of this econometric model that estimates systematic risk and risk premium in two passes allows us to determine the significance level of generatedmean-variance-efficient portfolios, using the tested CAPM.

The works of Urbański, Jawor and Urbański (2014), and Urbański (2015) prove that removal of speculative stocks and penny stocks from the analysis results in generating multifactor-efficient portfolios by selected ICAPM procedures.

The estimation of the cost of capital is also associated with the determination of the error value at the assumed level of significance. Therefore, I present a method to create a confidence interval of the cost of capital. Based on the adopted assumptions, the cost of capital is a product of the systematic risk and risk price. The risk price and systematic risk are determined as parameters of linear regression. In this linear regression model, the monthly return distributions are close to normal. Therefore, the distribution of the estimate of the parameters should be also close to normal. However, the distribution of cost of capital is unknown because it is a nonlinear function of

normally distributed estimates. In order to determine the distribution of the cost of capital and effectively evaluate its error I use the bootstrap method.

In the Section "Empirical literature and cost of equity capital using the CAPM", I present the methodologies for estimating the cost of capital using the CAPM applications. The Section "Data and Results" presents the data and procedures of portfolio construction, and the results of estimated values of risk price for different speculation and penny stocks boundary conditions (in Subsection "The expected value of the risk price"). Also, this Section shows pricing errors of the tested portfolios (in the Subsection "Pricing errors"), and distributions of estimated cost of capital and its bootstrapped confidence intervals for characteristic portfolios (in Subsection "Distributions of capital cost of modeled portfolios"). In the last Section, I draw conclusions.

EMPIRICAL LITERATURE AND COST OF EQUITY CAPITAL USING THE CAPM

The cost of the company's capital estimated on the basis of CAPM is based on the realized returns perceived by the external observer. Such an estimated cost of capital may reflect the cost of the investment project portfolio if the projects implemented are independent of each other and not loaded with real options. Also, such an estimated cost of capital may reflect the cost of the investment project portfolio ifthe impact of applied real options is small. In general, the estimation of the cost of capital, constituting the cost of the portfolio of the on-investment projects implemented by the company, requires adjusting the market returns for the impact of real options.

Estimating the cost of capital using the CAPM is basically limited to public companies. In the case of non-public companies, the model proposed by Hamada (1972) may be applied. This model combines CAPM with the Modigliani-Miller theorem. Other methods of estimating the cost of capital include the discounted cash flow method (DCF) or the method based on the yield on the company's own bonds with a risk premium (see: Brigham, Gapensky, 2000, p. 260). Prat (2002) as well as Fama and French (2002) argue thatthe DCF method consistently produces lower estimates of the cost of capital than CAPM. The capital cost estimated using the DCF method does not reflect shares corresponding to risk-free investments and risky investments.

The cost of capital estimated using the CAPM depends on the estimated systemic risk (beta) and the estimated risk premium. Betas can be estimated based on the excess returns (or total return) on the stock (portfolio) being tested relative to the excess returns (or total return) on the market index³. Ibbotson Associates estimates betas based on 60 monthly excess returns, and the 30-day Treasury bill for the risk free rate used to compute excess returns⁴. Other boundary conditions chosen arbitrarily are the length of the historical period of estimating the risk premium, and estimation method of the risk premium. Fernandez (2011) has analyzed the estimated risk premiums over the past 30 years on the basis of 150 finance textbooks. The estimated annual risk premium varied from 3% to 10%. Damodaran (2017) presents the US equity risk premium, using different approaches, in January 2013, with the lowest value being 3.20% and the highest of 6.02%. The research of Cornell (1999, 59-60) suggests that the risk premium is a non-stationary time series. Other studies on the US market show thatif the length of the measurement period is shortened, the standard error risk premium estimation increases (see: Prat, 2002, p. 63, and Mauboussin & Callahan, 2013, p. 12). It justifies the use of long historical periods. Ibbotson Associatesadvocates use of the full historical period covered by the data.

Most procedures for evaluating the cost of capital use the financial form of the CAPM in which the risk premium is calculated as the estimator of the expected value of the difference between the market return and the risk-free rate. The arithmetic mean is an unloaded and consistent estimator of the expected value. However, this procedure prevents the analysis of whether the classic CAPM generates mean-variance-efficient portfolios in the audited period of the tested market. In the light of Ferson and Locke (1998), the correct estimation of the confidence interval of the cost of capital is in this case very complicated and even impossible⁵.

The research studies that attempt to estimate the cost of capital of selected Polish companies with the use

³ Differences between betas estimated based on excess returns or total returns are small.

⁴ Ibbotson Associates is a financial software, data, consulting, and training firmheadquartered in Chicago, Illinois. Established in 1977 by Roger Ibbotson.

⁵ Due to the lack of information whether the capital cost estimation (on the tested market) using the CAPM is appropriate.

of the above-mentioned procedures were conducted, among others, by Ranosz and Kustra (2013), or Ranosz and Kowal (2016). The work of Kozarević and Džafić (2014) is an example of other research studies on the use of the CAPM to estimate the cost of capital on the Sarajevo Stock Exchange.

The financial form of the CAPMcan be expressed as:

$$E(r_i) = E(RF) + \beta_{iM}E(RM - RF), \quad (1$$

where E(RM-RF) is an expected excess market return over risk free asset, and $\theta_{_{IM}}$ is the systematic risk of stock i, due to the market portfolio. E(RF) is an expected return of risk free asset, and E(ri) is an expected return of the analyzed asset.

Then, the corresponding econometric model, for estimating the systematic risk θ_{iM} and expected systematic risk price E(RM-RF), can be presented in two passes in equations (2) and (3).

In the first pass as follows:

$$r_{it} - RF_t = \beta_{i0} + \beta_{iM}(RM_t - RF_t) + e_{it};$$

 $t = 1, \dots, T : \forall i = 1, \dots, m.$ (2)

In the second pass as follows:

$$r_{it} - RF_t = \gamma_0 + \gamma_M \widehat{\beta_{tM}} + \varepsilon_{it};$$

$$t = 1, \dots, T, t = 1, \dots, m.$$
(3)

The estimators γ_M and β_{iM} are used to determine the values of cost of capital (*Ccapi*) of analyzed stocks (portfolios) i, according to the equation (1). Usually, betas are estimated based on T=60 monthly periods. Thus, I estimate *Ccapi* according to Eq. (4)

$$Ccap_{i} = E(RF) + \gamma_{0} + \widehat{\beta_{iM}^{T}} \widehat{\gamma_{M}};$$

$$i=1, \dots, m.$$
(4)

The estimator $\widehat{eta_{iM}^T}$ in equation (4) is evaluated from the equation

$$r_{it} = \beta_{i0}^{T} + \beta_{iM}^{T} R M_{t} + s_{it};$$

 $t=1,...,T: \forall i=1,...,m.$
(5)

In order to assess the accuracy of estimating cost values I apply the bootstrap technique, by bootstrapping the residuals of models (2), (3) and (5).⁶

I test the two classic CAPM applications to estimate the risk price. In the first application portfolios are formed on *BV/MV* and capitalization (Size). The second CAPM application uses the following observation: "The

economic state variable that produces variation in the future earnings and returns related to size and *BV/MV* is a vector of structure of the past long-term differences in profitability" (Urbański et al., 2014, p. 84).

In this application portfolios are formed on the *NUM* and *DEN* functions of *FUN* functional.

The functional FUN is shown below.7

$$FUN = \frac{NUM}{DEN} = \frac{nor(ROE)*nor(AS)*nor(APO)*nor(APN)}{nor(MV/E)*nor(MV/BV)}, (6)$$

where

$$ROE = F_{1}; AS = F_{2} = \frac{\sum_{t=1}^{i} S(Q_{t})}{\sum_{t=1}^{i} \overline{S(nQ_{t})}};$$

$$APO = F_{3} = \frac{\sum_{t=1}^{i} PO(Q_{t})}{\sum_{t=1}^{i} \overline{PO(nQ_{t})}};$$

$$APN = F_{4} = \frac{\sum_{t=1}^{i} PN(Q_{t})}{\sum_{t=1}^{i} \overline{PN(nQ_{t})}};$$

$$MV/E = F_{5}; \quad MV/BV = F_{6}.$$
(7)

Variables F_j (j=1,...,6) are transformed to standardised areas ranging $\langle aj;bj \rangle$, in keeping with Eq. (8)

$$nor(\mathbf{F}_{j}) = \left[a_{j} + (b_{j} - a_{j}) * \frac{\mathbf{F}_{j} - c_{j} * \mathbf{F}_{j}^{\min}}{d_{j} * \mathbf{F}_{j}^{\max} - c_{j} * \mathbf{F}_{j}^{\min} + e_{j}} \right]. \tag{8}$$

In Equations (7) and (8), the corresponding indications are as follows: ROE is return on book equity; $\sum_{t=1}^{i} S(Q_t)$,

$$\sum_{t=1}^{i} PO(Q_t), \sum_{t=1}^{i} PN(Q_t)$$
 are values that are accumulated

from the beginning of the year as net sales revenue (S), operating profit (PO) and net profit (PN) at the end of "i"

quarter
$$(Q_i)$$
; $\sum_{t=1}^{i} \overline{S(nQ_t)}$; $\sum_{t=1}^{i} \overline{PO(nQ_t)}$;

 $\sum_{t=1}^{i} \overline{PN(nQ_t)}$ are average values, accumulated from the beginning of the year as S, PO and PN at the end of Q_i over the last n years (the present research assumes that n=3 years); BV is the book value, MV/E is the market-to-earning value ratio; E is the average earning for the last four quarters; MV/BV is the market-to-book value ratio; $a_{j'}$ $b_{j'}$ $c_{j'}$ $d_{j'}$ e_{j} are variation parameters. In equilibrium modeling F_j (j=1,...,6) can be transformed into an equal normalized area <1;2> (Urbański, 2011).

DATA AND RESULTS

The cost of capital is evaluated using two classic

 $^{^{\}rm 6}$ Bootstrap methods are widely described by Efron and Tibshirani (1993).

⁷ NUM and DEN are basic functions of the state functional modifying the classic Fama and French (1993) model and widely described in the works of Urbański (2011, 2012).

CAPM applications which are mentioned in the previous Section. In the case of the first pricing application the quintile portfolios are formed on BV/MV and Size, in two directions. Each BV/MV quintile is divided into new Size quintiles. In the case of the second application the quintile portfolios are formed on the NUM and DEN function, in two direction. Each NUM quintile is divided into new DEN quintiles. There are 25 portfolios tested altogether.

I analyze the monthly returns of the stocks listed on the WSE in 1995-2017. Data referring to the fundamental results of the inspected companies are taken from the database drawn up by Notoria Serwis Sp. z o.o. Data for defining returns on securities are provided by the WSE.

THE EXPECTED VALUE OF THE RISK PRICE

I analyze ten modes of samples for estimation of expected value of the risk price, with regard to the market portfolio. Mode 1 considers all WSE stocks, except companies characterized by a negative book value. In modes from 2 to 8 penny stocks, with market values lower than 0.5, 1.0, 1.5, 2.0, 3.0, 4.0 and 5.0 PLN are eliminated. Modes 9 and 10 examine hypothetical cases of the elimination of all speculative stocks with the assumed criteria. Mode 9 (indicated SPEC1) eliminates the companies that meet the following boundary conditions: 1) MV/BV > 100; 2) ROE < 0 and BV > 0; and 3) MV/BV > 30 and $r_{it} > 0$, Mode 10 (indicated SPEC2) eliminates speculative stocks, meeting additional condition

Table 1: The values of the risk price (y_M) estimated from second-pass regressions by the classic CAPM for portfolios formed on BV/MV and Size

	<u>~</u>			
$r_{it} - RF_t$	$= \gamma_0 + \gamma_M \widehat{\beta_{i,j}}$	$_{M}+\varepsilon_{it}; t=1$	252;	i=125

	Excluded penny stocks below (PLN)								Speculative stocks	
Parameter	0.0	0.5	1.0	1.5	2.0	3.0	4.0	5.0	SPEC1	SPEC2
	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
<i>y</i> ₀ , %	-4.13	-3.70	-3.14	-3.32	-2.94	-3.21	-1.94	-3.05	-15.96	-8.34
<i>t</i> -stat	-3.72	-3.42	-3.06	-3.17	-2.72	-2.69	-1.73	-2.30	-17.65	-10.66
SH <i>t</i> -stat	-3.26	-3.08	-2.85	-2.90	-2.55	-2.49	-1.69	-2.15	-6.94	-7.73
p-value, %	0.11	0.21	0.44	0.37	1.09	1.28	9.05	3.16	0.00	0.00
<i>y</i> _{<i>M</i>} %	3.82	3.34	2.73	3.03	2.55	2.81	1.41	2.63	16.14	6.56
<i>t</i> -stat	3.13	2.81	2.43	2.62	2.13	2.15	1.14	1.79	15.74	6.83
SH <i>t</i> -stat	2.78	2.56	2.29	2.43	2.01	2.01	1.12	1.68	6.72	5.21
p-value, %	0.54	1.04	2.22	1.51	4.45	4.45	26.46	9.23	0.00	0.00
GRS-F	4.01	4.10	4.03	3.69	3.62	3.47	2.97	3.59	24.36	13.10
<i>p</i> -value, %	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00
$Q^{\scriptscriptstyle A}(F)$	2.98	2.89	2.69	2.21	2.99	2.78	2.56	3.20	2.68	1.85
p-value, %	0.00	0.01	0.02	0.12	0.00	0.01	0.03	0.00	0.02	1.55
R_{LL}^2 , %	6.51	4.93	1.23	5.40	1.35	7.18	-0.03	3.71	48.51	49.24

Note: 252 monthly periods are analyzed from May 1995 through May 2017. RF_t is the 91-day Polish Treasury bill return. $\widehat{eta_{lM}}$ is the loading on the market factor estimated from first-pass time-series regressions. GRS-F is F-statistic of Gibbons, Ross and Shanken (1989). $Q^A(F)$ reports F-statistic and its corresponding p-value indicated below in brackets for Shanken's (1985) test that the pricing errors in the model are jointly zero. SH t-stat is Shanken's (1992) statistic adjusting for errors-in-variables. Following Lettau and Ludvigson (2001) R_{LL}^2 is a measure that shows the fraction of the cross-sectional variation in average returns that is explained by each model and is calculated as follows: $R_{LL}^2 = [\sigma_c^2(\bar{r_i}) - \sigma_c^2(\bar{\epsilon_i})]/\sigma_c^2(\bar{r_i})$ where σ_c^2 denotes a cross-sectional variance, and variables with bars above denote time-series averages. The Prais-Winsten procedure for correction of first-lag autocorrelation is used. SPEC1 eliminates speculative stocks meeting one of the following boundary conditions: 1) MV/BV > 100; 2) ROE < 0 and BV > 0; and 3) MV/BV > 30 and $r_i > 0$, where BV > 0 is return on book value BV > 0, where BV > 0 is return of portfolio BV > 0 and BV

Source: Own research

4) MV/E< 0.

In Tables 1 and 2, I show the estimated values of the risk price, estimated from the second-pass regressions (3) for eliminated penny stocks and speculative stocks. None of the test sample does not generate mean-variance-efficient portfolios. The systematic market risk is not priced for portfolios formed on BV/MV and Size, except in the case of Mode SPEC1. This is evidenced by SH t-statistic adjusting for errors-in-variables. If speculative stocks SPEC1 are eliminated: SH t-stat=3.32 (with corresponding p-value=0.09%). However, intercept is significant at a high level below 0.002%. This means that the market factor itself is not a correct determinant of returns.

The CAPM application, if portfolios are formed on NUM and DEN, generates a more appropriate pricing of systematic risk.If penny stocks below 3.00 PLN are excluded the risk price is about 2.8% per monthand is significant in each case at the level below 5%. However, the cross-sectional determination coefficient takes low values below 8%. Also, the intercepts are significant, assuming values about -3% with correspondingp-values below 1.28%, after adjusting for errors-in-variables. The hypothetical modes SPEC1 and SPEC2 generate significant extremely high-risk prices of about 16% and 7% per month.

The coefficient takes high values about 50%. However, for these cases the intercepts also are significant, assuming high negative values -16% and -8% with corresponding p-values below 0.1%.

PRICING ERRORS

In Figures 1 and 2, I show the visual assessment of both tested CAPM applications.⁸ These figures present pricing errors of the tested portfolios. I exclude portfolios whose pricing errors are extreme (below the 1.25th percentile or above 98.75th percentile in the cross section). Portfolios are marked with numbers 1 to 25. In Fig. 1 portfolios 1 to 5 are formed on the highest *NUM* values, from 21 to 25 on the lowest *NUM* values. Portfolios: 1, 6, 11, 16 and 21 are formed on the smallest *DEN*, and portfolios: 5, 10, 15 20 and 25 are formed on the highest *BV/MV* values, from 21 to 25 on the lowest *BV/MV* values. Portfolios: 1, 6, 11, 16 and 21 are formed on the smallest Size, and portfolios: 5, 10, 15 20 and 25 are formed on the biggest

Table 2: The values of the risk price vector (y) estimated from second-pass regressions by the classic CAPM for portfolios formed on *NUM* and *DEN*

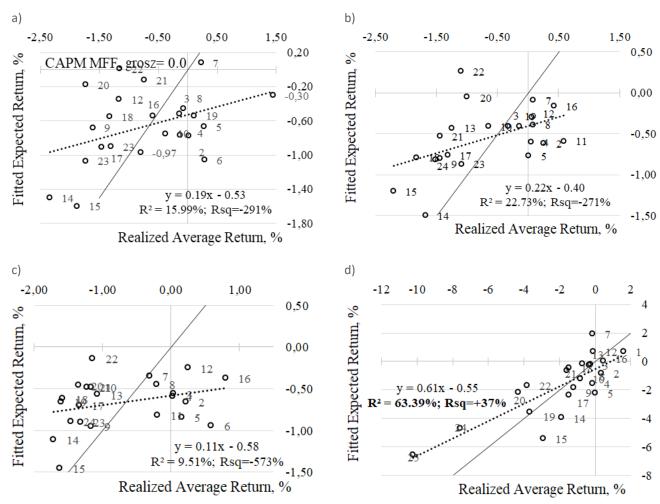
$$r_{it} - RF_t = \gamma_0 + \gamma_M \widehat{\beta_{i,M}} + \varepsilon_{it}$$
; $t=1.....252$; $i=1.....25$

	Excluded penny stocks below (PLN)								Speculative stocks	
Parameter	0.0	0.5	1.0	1.5	2.0	3.0	4.0	5.0	SPEC1	SPEC2
	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
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y _M , %	3.82	3.34	2.73	3.03	2.55	2.81	1.41	2.63	16.14	6.56
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<i>p</i> -value, %	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00
$Q^{\scriptscriptstyle A}(F)$	2.98	2.89	2.69	2.21	2.99	2.78	2.56	3.20	2.68	1.85
<i>p</i> -value, %	0.00	0.01	0.02	0.12	0.00	0.01	0.03	0.00	0.02	1.55
R_{LL}^2 , %	6.51	4.93	1.23	5.40	1.35	7.18	-0.03	3.71	48.51	49.24

Note: See Tab. 1.
Source: Own research

⁸ The visual assessment of pricing models is proposed by Jagannathan and Wang (1996).

Figure 1: Fitted expected returns versus realized average returns simulated by the classic CAPM for portfolios formed on *NUM* and *DEN*: a) Negative-*BV* stocks are excluded from the portfolios, b) penny stocks below 1.50 PLN are excluded, c) penny stocks below 3.00 PLN are excluded, d) SPEC1 stocks are excluded



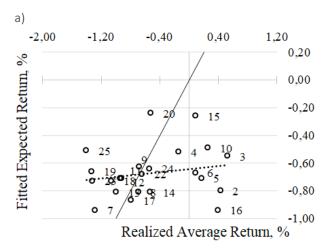
Note: The figure shows the pricing errors for each of the 25 portfolios. Each scatter points represents one portfolio. For each portfolio i, the realized average return is the time-series average of the portfolio returns. The fitted expected return is the value for the expected return $E[r_i]$, in the following regression model: $E[r_i] = \gamma_0 + \gamma_M \beta_{iM'}$, where β_{iM} is the systematic market risk estimated in the first-pass GLS regression of the returns' excess of the portfolios in respect of the market factor, γ_0 is the expected return on a "zero-beta" portfolio, γ_M is the market risk price, γ_0 and γ_M are estimated in the second-pass GLS regression. If the model fit perfectly, all the points would lie along the 45-degree line. Rsq is R² statistics measure of the success of the regression in predicting the values of the fitted expected return against their realized average returns if the regression does not have an intercept and contains loading restriction equal one (45-degree line). The broken line and R² represent the actual regression. SPEC1 eliminates speculative stocks meeting one of the following boundary conditions: 1) MV/BV > 100; 2) ROE < 0 and BV > 0; and 3) MV/BV > 30 and $r_i > 0$, where MV is stock market value, ROE is return on book value (BV), r_i is return of portfolio i during period t.

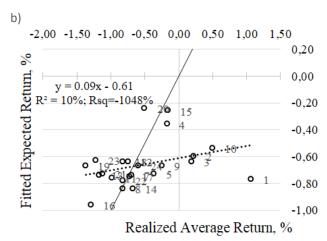
Source: Own research

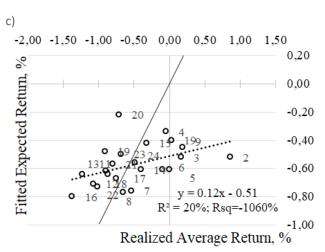
Size. If the model fits perfectly, all the points should lie along the 45-degree line. Rsq is R² statistics measure of the success of the regression in predicting the values of the fitted expected return against their realized average returns if the regression does not have an intercept and contains loading restriction equaling one (45-degree line). The broken line and R² represent the actual regression.

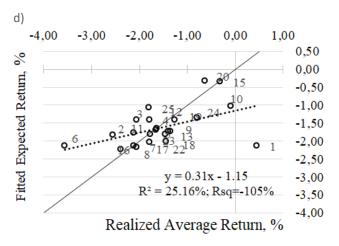
On the basis of the obtained results one should note that if negative-*BV* stocks or penny stocks below 4.00 PLN are excluded, both tested CAPM applications generate great pricing errors for Modes from 1 to 8. The Rsq coefficient assumes high negative values. The errors are slightly lower for CAPM tested on the portfolios formed on *NUM* and *DEN*. The elimination of penny stocks

Figure 2: Fitted expected returns versus realized average returns simulated by the classic CAPM for portfolios formed on BV/MV and Size: a) Negative-BV stocks are excluded from the portfolios, b) penny stocks below 1.50 PLN are excluded, c) penny stocks below 4.00 PLN are exclude, d) SPEC1 stocks are excluded









Note: See Fig. 1.
Source: Own research

does not significantly affect the size of pricing errors. In the case of excluding speculative stocks SPEC1 the pricing errors significantly decrease. The Rsq coefficient is positive and equals to 37%, for portfolios formed on *NUM* and *DEN*. If the portfolios are formed on *BV/MV* and Size, the Rsq coefficient increases from about -1100% to -105%. However, prices of SPEC1 and SPEC2 stocks vary considerably. Urbański et al. (2014) present the percentage of speculative stocks depending on the price, at the beginning of 64 quarters in 1996-2011. The largest number of speculative stocks is in the range of prices (1.00 PLN;2.00 PLN). The number of all SPEC1 stocks in relation to all listed stocks accounts for 11.73%, and for 12.75% in the case of SPEC2 stocks.

Doing an in-depth analysis of the risk price estimation results, rejecting the hypothetical implementations SPEC1 and SPEC2, I decided that a deeper analysis should be

focused on the case in which penny stocks below 1.50 PLN are excluded.

DISTRIBUTION OF CAPITAL COST OF MODELED PORTFOLIOS

The cost of capital is estimated in two variants. In Variant 1 the betas are estimated on the basis of the last 120 months, from period 133 to period 252. In Variant 2, the procedure similar to the one proposed by Zhi Da et al. (2012) is applied, and the betas are estimated using (*t*-61, *t*-1) a sixty-month rolling window, with a rolled step of one month, using the whole tested period of 252 months.⁹

Table 3 shows the statistics of normality tests of

⁹ Zhi Da et al. (2012, p. 212) use delayed betas for 24 months.

Table 3: Statistics of normality tests of bootstrapped cost of capital, risk price and systematic risk, estimated by the classic CAPM, for portfolio formed on the highest BV/MV and the smallest Size

Davanatan	Normality test statistic (p-value)							
Parameter	Doornik-Hansen	Shapiro-Wilk	Lilliefors	Jarque'a-Bera				
Cost of capital	3201.77	0.9358	0.0879	8248.6				
Variant 1	(0)	(4.57e-054)	(0)	(0)				
Risk price $\widehat{Y_M}$	1.6291	0.9999	0.0043	1.5490				
	(0.4428)	(0.8696)	(0.96)	(0.4609)				
Systematic risk - Variant 1 $\widehat{eta_{1,M}}$	502.27	0.9835	0.0423	1154.37				
	(8.57e-110)	(1.80e-032)	(0)	(2.14e-251)				
Cost of capital	47.3488	0.9986	0.0129	59.0637				
Variant 2	(5.23e-011)	(6.48e-008)		(1.49e-013)				
Systematic risk - Variant 2 $\widehat{eta_{1,M}}$	3.0287	0.9998	0.0055	3.0508				
	(0.2200)	(0.3947)	(0.65)	(0.2154)				

Note: I analyze stock companies registered on the Warsaw Stock Exchange in the period May 1995 through May 2017 that were showing a positive *BV* and with market prices not lower than 1.50 PLN. The risk price is estimated by regression (3) using 252 monthly periods, while betas based on regression (5). In Variant 1 betas are estimated on the basis of the last 120 months, from the 133 to 252 period. In Variant 2 betas are estimated on the basis of a sixty-months rolling window, with a rolled step of one month, using the whole tested period of 252 months. The bootstrap procedure is based on 10000 data resamples.

Source: Own research

bootstrapped cost of capital, bootstrapped risk price and systematic risk, estimated by the classic CAPM, for a portfolio formed on the highest *BV/MV* and the smallest Size values.

Table 4 shows the statistics of normality tests of bootstrapped cost of capital, bootstrapped risk price and systematic risk estimated by the classic CAPM, for a portfolio formed on the highest NUM and the smallest DEN values.

Figure 3 shows histograms of bootstrapped cost of capital estimated by the classic CAPM for a portfolio

formed on NUM and DEN. Figure 4shows histograms of bootstrapped cost of capital estimated by the classic CAPM for a portfolio formed on BV/MV and Size.¹⁰

The cost of capital for a portfolio formed on the highest BV/MV and the smallest Size, estimated in both Variants, does not show normality of distribution. However, in the case of estimation in Variant 2, the cost of capital distribution, for a portfolio formed on the highest

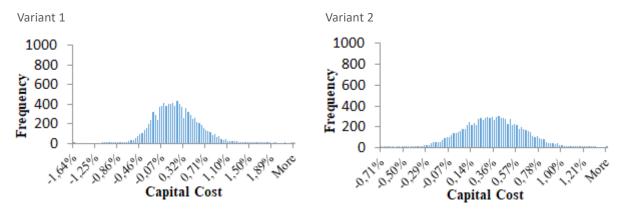
Table 4: Statistics of normality tests of bootstrapped cost of capital, risk price and systematic risk, estimated by the classic CAPM, for a portfolio formed on the highest *NUM* and the smallest *DEN*

Parameter	Normality test statistic (p-value)							
	Doornik-Hansen	Shapiro-Wilk	Lilliefors	Jarque'a-Bera				
Cost of capital	352.354	0.9870	0.0368	626.729				
Variant 1	(3.07e-077)	(3.19e-029)		(8.08e-137)				
Risk price $\widehat{\gamma_M}$	7.7065	0.9997	0.0080	7.8974				
	(0.0212)	(0.1783)	(0.12)	(0.0193)				
Systematic risk - Variant 1 $\widehat{eta_{1,M}}$	0.7855	0.9999	0.0087	0.8399				
	(0.6752)	(0.8570)	(0.06)	(0.6571)				
Cost of capital	2.7094	0.9998	0.0057	2.7393				
Variant 2	(0.2583)	(0.4730)	(0.6)	(0.2542)				
Systematic risk - Variant 2 $\widehat{\beta_{1,M}}$	2.7420	0.9997	0.0071	2.7073				
	(0.2539)	(0.2117)	(0.24)	(0.2583)				

Note: See Tab. 3. Source: Own research

¹⁰ The histograms of the bootstrapped market risk price and systematic market risk are available from the author on request.

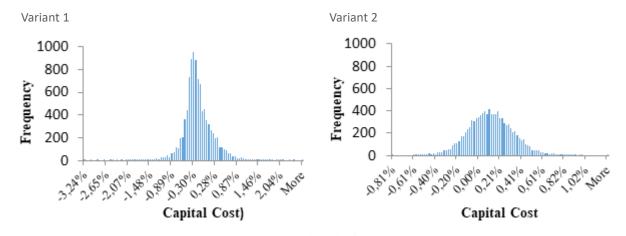
Figure 3: Histograms of bootstrapped cost of capital of portfolio 1 formed on the highest NUM and the smallest DEN



Note: I analyze stock companies registered on the WSE in the period May 1995 through May 2017 that were showing a positive BV and with market prices not lower than 1.50 PLN. The risk priceis estimated using 252 monthly periods. In Variant 1 betas are estimated on the base of the last 120 months, from the 133 to 252 period. In Variant 2 betas are estimated on the base of a sixty-months rolling window, with a rolled step of one month, using the whole tested period of 252 months. The bootstrap procedure is based on 10000 data resamples.

Source: Own research

Figure 4: Histograms of bootstrapped cost of capital of portfolio 1 formed on the highest NUM and the smallest DEN



Note: See Fig. 3. Source: Own research

NUM and the smallest DEN, is normal.

Distributions of risk price $\widehat{\gamma_M}$, estimated using the portfolios formed on BV/MV and Size, and on NUM and DEN are normal.

Distributions of systematic risk $\widehat{\beta_{1,M}}$ for a portfolio formed on the highest BV/MV and the smallest Size, estimated in Variant 1, does not show normality of distribution. While, distributions of $\widehat{\beta_{1,M}}$ for these portfolios, estimated in Variant 2, are normal.

Distributions of systematic risk $\widehat{\beta_{1,M}}$ for a portfolio formed on the highest *NUM* and the smallest *DEN*, estimated in both Variants, are normal.

The above conclusions are confirmed by the results of the four normality tests (see: Tab. 3 and Tab. 4).¹¹

In Table 5 I show the estimated values of the cost of capital by the classic CAPM, for 25 model portfolios, on the basis of Variant 2.¹² In the first pass, an econometric model (2) to estimate the systematic risk is used. In the second pass, model (3) to estimate the risk price is used. The current values of betas, for cost of capital calculation, are estimated by regression (5).

Figure 5 shows distributions of cost of capital and the corresponding bootstrapped 95 confidence intervals for 25 model portfolios.

 $^{^{\}rm 11}$ The results of the normality tests for the all portfolios are available from the author on request.

¹² The capital cost values estimated on the basis of Variant 1 are available from the author on request.

Panel A: Portfolios formed on BV/MV and Size

I pass: r_{it} - RF_t = $\theta_{i,0}$ + θ_{iM} (RM_t - RF_t) + e_{it} ; t =1.....252; \forall i =1.....25 II pass: r_{it} - RF_t = y_0 + y_M $\widehat{\beta_{iM}}$ + e_{it} ; t =1.....252; i = 1.....25 Average betas of asset i: r_{it} = a_i + β_{iM}^T RM_t + e_{it} ; t =1.....60 : 193...252 y_0 = -0.0239; SH t-stat=-1.8506 Ccapi = E(RF) + $\widehat{\gamma_0}$ + $\widehat{\beta_{iM}^{aw}}\widehat{\gamma_M}$; β_{iM}^{aw} = $\frac{1}{193}\sum_{t=1}^{193}\widehat{\beta_{iM}^{ft}}$

Book to market value BV/MV portfolios									
61 15 11	Low Growth	2	3	4	High Value				
Size portfolios		Monthly median values							
Small (95 conf. interv.)	Portfolio 21) -0.05 (-0.24÷0.14)	Portfolio 16) -0.19 (-0.48÷0.08)	Portfolio 11) -0.14 (-0.37÷0.08)	Portfolio 6) -0.06 (-0.37÷0.30)	Portfolio 1) 0.12 (-0.27÷0.54)				
2 (95 conf. interv.)	-0.15 (-0.38÷0.07)	-0.11 (-0.30÷0.09)	-0.05 (-0.24÷0.15)	0.02 (-0.22÷0.29)	0.17 (-0.29÷0.64)				
3 (95 conf. interv.)	-0.06 (-0.24÷0.13)	0.03 (-0.22÷0.31)	-0.10 (-0.30÷0.09)	-0.11 (-0.32÷0.08)	0.11 (-0.26÷0.48)				
4 (95 conf. interv.)	-0.10 (-0.30÷0.09)	-0.14 (-0.37÷0.07)	-0.09 (-0.28÷-0.10)	0.03 (-0.22÷0.30)	0.20 (-0.32÷0.74)				
Big (95 conf. interv.)	Portfolio 25) -0.05 (-0.23÷0.15)	Portfolio 20) 0.17 (-0.29÷0.64)	Portfolio 15) 0.09 (-0.25÷0.44)	Portfolio 10) -0.03 (-0.22÷0.18)	Portfolio 5) -0.25 (-0.64÷0.11)				

Panel B: Portfolios formed on NUM and DEN

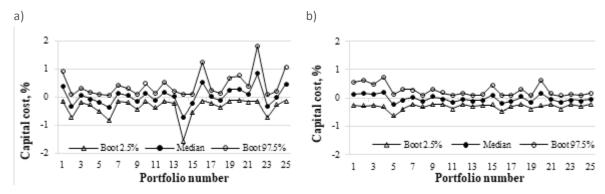
$$\begin{split} \text{I pass: } r_{it} - RF_t &= \theta_{i,0} + \theta_{ilM} \left(RM_t - RF_t \right) + e_{it'} \ t = 1.....25; \ \forall \ i = 1.....25 \\ \text{II pass: } r_{it} - RF_t &= y_0 + y_M \ \widehat{\beta_{iM}} + e_{it'} \ t = 1.....252; i = 1.....25 \\ \text{Average betas of asset } i : r_{it} &= a_i + \beta_{iM}^T RM_t + e_{it'} \ t = 1.....60 : 193...252 \\ y_0 &= -0.0239; \ SH \ t + \text{stat} = -1.8506 \\ Ccapi &= E(RF) + \widehat{\gamma_0} + \widehat{\beta_{iM}^2} \widehat{\gamma_M}; \quad \beta_{iM}^{aw} &= \frac{1}{193} \sum_{t=1}^{193} \widehat{\beta_{iM}^2} \end{split}$$

NUM portfolios Dynamics of increase of financial results								
DEM montfolion	Low	2	3	4	High			
DEN portfolios		Mo	onthly median values					
Small/Cheap (95 conf. interv.)	Portfolio 21) 0.09 (-0.17÷0.37)	Portfolio 16) 0.54 (-0.13÷1.24)	Portfolio 11) -0.12 (-0.39÷0.12)	Portfolio 6) -0.38 (-0.83÷0.06)	Portfolio 1) 0.38 (-0.13÷0.91)			
2 (95 conf. interv.)	0.82 (-0.16÷1.83)	0.00 (-0.23÷0.23)	0.17 (-0.14÷0.53)	0.12 (-0.15÷0.41)	-0.33 (-0.74÷0.07)			
3 (95 conf. interv.)	-0.33 (-0.74÷0.07)	-0.12 (-0.37÷0.12)	-0.01 (-0.24÷0.22)	0.05 (-0.18÷0.31)	0.06 (-0.18÷0.32)			
4 (95 conf. interv.)	-0.04 (-0.28÷0.19)	0.26 (-0.13÷0.69)	-0.73 (-1.56÷0.08)	-0.16 (-0.44÷0.10)	-0.05 (-0.28÷0.18)			
Big/Priced (95 conf. interv.)	Portfolio 25) 0.44 (-0.13÷1.06)	Portfolio 20) 0.29 (-0.12÷0.75)	Portfolio 15) -0.23 (-0.56÷0.08)	Portfolio 10) 0.14 (-0.15÷0.47)	Portfolio 5) -0.19 (-0.50÷0.09)			

Note: Stock companies listed on the WSE from May 1995 through May 2017 that are showing a positive BV and with market prices not lower than 1.50 PLN are analyzed. In Panel A quintile portfolios are formed on BV/MV and each of them is divided in the increasing manner into five on Size. In Panel B quintile portfolios are formed on NUM and each of them is divided in the increasing manner into five on DEN. The corresponding 95 confidence intervals appear in brackets. The market risk price (gamma) is estimated using 252 monthly periods. The systematic market risk (beta) is estimated using a sixty-months rolling window, with rolled step of one month. The lower and the upper limit of the confidence intervals are calculated using the bootstrap distributions of the estimated beta and gamma with 10000 bootstrap iterations.

Source: Own research

Figure 5: Cost of capital values for 25 model portfolios and the corresponding 95 confidence intervals obtained using the classic CAPM



Note: Quintile portfolios are formed on NUM and DEN in Fig. a) and on BV/MV and Size in Fig. b). Portfolios 1-5 are characterized the highest value of NUM (BV/MV) and portfolios 21-25 are characterized the lowest value of NUM (BV/MV). Portfolios 1. 6. 11. 16 and 21 are characterized the smallest value of DEN (Size). Portfolios 5. 10. 15. 20 and 25 are characterized the biggest value of DEN (Size). Boot 2.5% and Boot 97.5% represent the lower and the upper limit of the bootstrap generated confidence intervals. The lower and the upper limit of the confidence intervals are calculated using 10000 bootstrap iterations.

Source: Own research

Panel A in Table 5 presents the values of cost of capital, for portfolios formed on *BV/MV* and Size, estimated by the classic CAPM. Estimates of bootstrapped 95% confidence interval change from negative to positive for all portfolios. However, the median confidence intervals achieve positive values for four portfolios (portfolios: 4, 9, 15 and 20) formed on the two biggest Size quintiles. Also, positive values of the median appear for the five value portfolios (portfolios: 1, 2, 3, 4 and 9 of the fourth and fifth *BV/MV* quintiles). The above results seem to confirm the well-known literature anomaly of growing returns for value portfolios. Also, the obtained results can be regarded as consistent with the work of Graham and Harvey (2001) who claim that large companies more frequently estimate cost of capital by CAPM.

Panel B in Table 5 presents the values of cost of capital, for portfolios formed on *NUM* and *DEN*, estimated by the classic CAPM. Estimates of bootstrapped 95% confidence interval also change from negative to positive for all portfolios. The median confidence intervals achieve positive values for seven portfolios (portfolios: 1, 7, 11, 12, 16, 21 and 22) formed on the first two *DEN* quintiles. The first *DEN* quintile contains portfolios characterized by high *BV/MV* and high earning-to-market value ratio. It means that the first *DEN* quintile contains value portfolios. Just as in panel A, these results confirm the anomaly of growing returns for value portfolios.

Negative and positive values of cost of capital can be explained by the following reasoning. A company is noticed by investors as a portfolio of projects and by their real options. The company stock returns are influenced by information reaching investors about the possibility of implementing such a perceived portfolio. In the case of risk price estimated by the tested CAPM applications, the cost of capital is under influence market returns, and takes into account the impact of real options. The cost of capital estimated in this way is influenced by all market information and may not reflect the actual cost of the company's projects. In other words, the negative values of cost of capital, estimated by the CAPM, do not constitute negative value of the company's projects. The obtained results allow for the following conjectures: if the estimated cost of capital applies to projects with real options then the estimated cost of capital, by the classic CAPM can be evaluated too highly for value portfolios i.e. for portfolios formed on high BV/MV or small DEN. In addition, the cost of capital can be estimated as too low for growth portfolios i.e. for portfolios formed on low BV/MV or big DEN, so they can charge the actual cost of capital of the company's projects.

CONCLUSIONS

My work is an attempt to estimate the cost of capital for the characteristic portfolios of stocks listed on the WSE. For this purpose, I use two classic CAPM applications. From the entire sample of 595 stocks, I eliminate penny stocks with prices below PLN 0.50, 1.00, 1.50, 2.00, 3.00,

The cost of equity capital in stock portfolios listed on the Warsaw Stock Exchange using the classic CAPM

4.00 and 5.00. In deeper analysis penny stocks below 1.50 PLN are excluded. However, the applied CAPM applications do not generate mean-variance-efficient portfolios despite the elimination of penny stocks. In order to estimate the confidence interval of the cost of capital I use the bootstrap method. The estimated cost of capital, based on stock returns, applies to a hypothetical portfolio of investment projects of a company, perceived by an external investor. The portfolio of company projects seen in this way is a combination of implemented and intended projects weighted with chosen real options. The estimated cost of capital, according to the applied procedure, can be seen as a predictor for investors. However, it cannot be a benchmark indicator for making decisions on the implementation of new investment projects of companies.

The conducted research leads to the following conclusions:

- 1) The cost of capital (perceived by an external investor) estimated according to the presented method refers to the company's investment project portfolio and opened position for real options on these projects.
- 2) The estimated cost of capital, according to the adopted procedure, assumes positive values for four

value quintiles (formed on high BV/MV), varying from 0.12%±0.39% to 0.20±0.53%% monthly. The average width of the confidence interval of cost of capital in these portfolios is about 0.88%.

- 3) The estimated cost of capital assumes negative values for five growth quintiles (formed on low BV/MV), varying from -0.05%±0.19%% to -0.15±0.23% monthly. The average width of the confidence interval of cost of capital in these portfolios is about 0.39%.
- 4) The estimated cost of capital assumes positive values for most portfolios formed on the first two DEN quintiles, varying from 0.09%±0.26%% to 0.82±0.98% monthly. The average width of the confidence interval of cost of capital in these portfolios is about 1.03%.
- 5) The tested CAPM application using the WSE stocks does not allow for generating mean-varianceefficient portfolios, which increases the cost of capital estimation errors.
- 6) For the purpose of estimating the cost of capital of the company's projects without real options, the components of the option-adjusted risk price, and optionadjusted systematic risk should be designated.

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